

Explicit Model Realizing Parton-Hadron Duality

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We present a model that realizes both resonance-Regge (Veneziano) and parton-hadron (Bloom-Gilman) duality. We first review the features of the Veneziano model and we discuss how parton-hadron duality appears in the Bloom-Gilman model. Then we review limitations of the Veneziano model, namely that the zero-width resonances in the Veneziano model violate unitarity and Mandelstam analyticity. We discuss how such problems are alleviated in models that construct dual amplitudes with Mandelstam analyticity (so-called DAMA models). We then introduce a modified DAMA model, and we discuss its properties. We present a pedagogical model for dual amplitudes and we construct the nucleon structure function $F_2(x, Q^2)$. We explicitly show that the resulting structure function realizes both Veneziano and Bloom-Gilman duality.

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I. INTRODUCTION

The subject of this paper was inspired to a large extent by the paper of Bjorken and Kogut [1], which argued that the dynamics of the strong interaction should be continuous across all regions of energy and momentum transfer. Thus one searches for a unified description of scattering processes that can explain the properties of reactions over a wide range of energies and momentum transfers. A qualitative picture of this is shown in the "road map" depicted in Fig. 1. The upper icon shows the behavior of the structure function $F_2(x, Q^2)$ vs. Bjorken x variable. At low energies the high- x region is dominated by inelastic resonances, whereas at high energies the resonances disappear and are replaced by a power-law behavior.

A classic example of a unified description of scattering is the so-called "Veneziano duality" [2]. The Veneziano model, reviewed in Sec II A, can be expanded in terms of a series of narrow resonances, and at high energies this amplitude demonstrates Regge behavior. This is shown schematically in the lower panel of Fig. 1. The Veneziano model has subsequently been extended to incorporate broad resonances and Mandelstam analyticity [3, 4].

Another interesting and important observation is the duality observed in deep inelastic scattering (DIS). At low energies and low virtualities these reactions are described in terms of hadronic properties, while at high virtuality such processes have a partonic description. At low energies the reactions are characterized by excita-

tion of nucleon resonances and are described in terms of hadronic excitations. At high energies a partonic description is more relevant and one sees a smooth curve in the scaling region. However, the smooth high-energy scaling curve essentially reproduces the average of the resonance peaks seen at low energies. This was first noted by Bloom and Gilman [5] by analyzing data from SLAC, and has since been confirmed by many measurements, with most recent from JLab [6]. The Bloom-Gilman duality region covers the right-hand side in the lower panel of Fig. 1. This phenomenon has been studied by several groups [7–13]. In particular, a relation between the amplitudes measured in exclusive lepto-production and the quark content of the nucleon was elucidated in Ref. [14]. However, for parton-hadron duality one is still lacking an explicit expression in terms of an amplitude like that of Veneziano. In this paper we propose an explicit model that realizes both resonance-Regge ("Veneziano") and parton-hadron ("Bloom-Gilman") duality.

We adopt a two-component model such as the one employed by Harari and Rosner [15]. The Harari-Rosner picture is characterized by a smooth background described by a direct-channel exotic trajectory, which at high energies is dual to the vacuum exchange amplitude (*i.e.*, the Pomeron trajectory) in the t channel. Separation and identification of the two components is not an easy task, but fortunately there exist reactions, such as J/Ψ photo- and electro-production, in which the scattering amplitude (or the structure function) is dominated by the diffractive component, as discussed in [16]. In the present paper we will be interested in the non-diffractive (resonance) component of the dual, resonance amplitude.

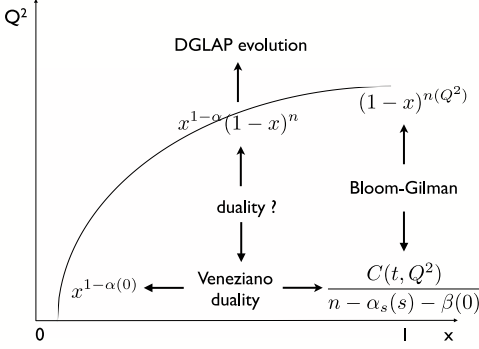
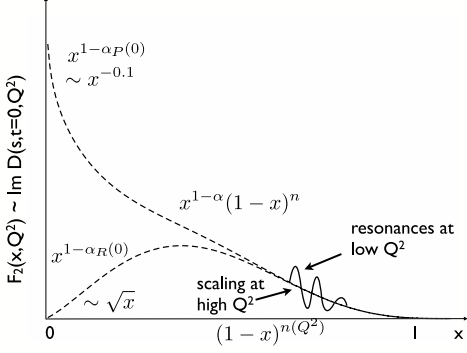


FIG. 1: Road map visualizing parton-hadron duality for the structure function $F_2(x, Q^2)$ at large x (upper panel) and in the whole x, Q^2 plane (lower panel).

Our main result can be summarized by the following formula which relates the F_2 structure function at low- Q^2 to the resonance expansion of a Veneziano-type amplitude and at high- Q^2 to the parton model,

$$\sum_n \frac{[f(Q^2)]^n \text{Im} \alpha_s(s(x, Q^2))}{[n - \text{Re} \alpha_s(s(x, Q^2))]^2 + \text{Im} \alpha_s(s(x, Q^2))^2} \begin{matrix} \uparrow Q^2 \rightarrow 0 \\ F_2(x, Q^2) \\ \downarrow Q^2 \rightarrow \infty \\ (1-x)^{n(Q^2)}. \end{matrix} \quad (1)$$

The parton model limit is characterized by the exponent $n(Q^2)$ and the hadronic limit by a function of the intercepts, α_s of s channel trajectories. For the sake of simplicity we ignore spin dependence in this paper. A fully consistent treatment of the scattering problem would require us to account for the spin dependence. However,

our goal here is to demonstrate qualitatively a proposed new way of constructing a "two-dimensionally dual" amplitude.

Our paper is organized as follows. In Sec. II A we review the narrow-resonance dual Veneziano model. Despite the qualitative successes of this picture, the zero-width resonances in this model are manifestly non-unitary and violate analyticity. In Sec. II B we show how these problems can be avoided by incorporating broad resonances via complex Regge trajectories and Mandelstam analyticity (the so-called DAMA picture). In Sec. II C, following Ref. [17], the DAMA model is extended off mass shell to make it applicable to deep inelastic lepton-hadron scattering. Based on this new dual amplitude, which we call Modified DAMA (or M-DAMA), a structure function $F_2(x, Q^2)$ is calculated in Sec. III. There we also show that the M-DAMA amplitude, and the related structure function, explicitly realize parton-hadron duality. Numerical examples and duality tests are presented in Sec. III B. A summary and discussion of the results can be found in Sect. IV.

II. DUAL MODELS

In this Section we summarize various dual models that attempt to provide a unified description of scattering phenomena over widely differing regions of energy and momentum transfer.

A. Narrow-resonance approximation: the Veneziano amplitude

Here we briefly review the basic features of resonance-Regge duality, which is explicitly realized in the Veneziano model [2]. The Veneziano amplitude is given by

$$V(s, t) = \int_0^1 dz z^{-\alpha(s)} (1-z)^{-\alpha(t)} = B(1-\alpha(s), 1-\alpha(t)) = \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}. \quad (2)$$

The amplitude $V(s, t)$ can be decomposed in a series of resonance poles:

$$V(s, t) = \sum_{n=1}^{\infty} \frac{C_n(t)}{n - \alpha(s)}, \quad (3)$$

where

$$C_n(t) = \frac{\Gamma(n + \alpha(t) + 1)}{n! \Gamma(\alpha(t) + 1)}. \quad (4)$$

Using the Stirling formula we can find the asymptotic behavior of $V(s, t)$,

$$V(s, t)|_{\alpha(s) \rightarrow \infty} \rightarrow [-\alpha(s)]^{\alpha(t)} \Gamma(1 - \alpha(t)) \quad (5)$$

Now, for small $|t|$ the Γ function varies slowly compared with the exponential one. Therefore, taking into account that the Veneziano model requires linear Regge trajectories, the amplitude $V(x, t)$ displays Regge asymptotic behavior,

$$V(s, t) \sim s^{\alpha(t)}. \quad (6)$$

The basic properties of the Veneziano model hold only in the narrow-resonance approximation, from Eq. (3) one has an infinite number of zero-width resonances. At high energies this produces real and linear Regge trajectories. As a result the original Veneziano model satisfies neither analyticity nor unitarity. To remedy the problems of an infinite number of narrow resonances, non-unitarity and an amplitude that lacks an imaginary part, a generalization of the Veneziano model was proposed, called dual amplitudes with Mandelstam analyticity or DAMA [3]. In the next section we will discuss the properties of the DAMA model.

B. Dual amplitude with Mandelstam analyticity (DAMA)

The so-called dual model with Mandelstam analyticity (DAMA) was proposed as a generalization of narrow-resonance dual models such as the Veneziano model. The DAMA model was introduced to avoid the manifest non-unitarity of narrow-resonance dual models [3]. In contrast to narrow-resonance dual models, DAMA requires non-linear, complex Regge trajectories. The dual properties of DAMA were studied in Ref. [4]. The DAMA amplitude [3] is given by,

$$D(s, t) = \int_0^1 dz \left(\frac{z}{g} \right)^{-\alpha_s(s')-1} \left(\frac{1-z}{g} \right)^{-\alpha_t(t'')-1}, \quad (7)$$

where $\alpha_s(s)$ and $\alpha_t(t)$ are Regge trajectories in the s and t channel respectively and we introduce the notation $x' = x(1-z)$, $x'' = xz$ (where $x = s, t, u$). In Eq. (7), $g > 1$ is a parameter.

The introduction of the integration variable accompanying the Mandelstam variable in the power of Eq. (7) enables (moreover, necessitates!) the use of nonlinear, complex Regge trajectories (impossible in the Veneziano model), required by unitarity and analyticity. The specific form of these functions, called *homotopies*, map the physical trajectory onto a linear function. It is known that unitarity (and its violation) is particularly sensitive to the singularity structure of the amplitude. DAMA as

defined above, has the pole structure, threshold singularities and the boundary of the double spectral function required by unitarity. For more details see [3].

In the limit of $s \rightarrow \infty$ and fixed t the DAMA model exhibits Regge behavior,

$$D(s, t) \sim s^{\alpha(t)}. \quad (8)$$

In the resonance region, the pole structure of DAMA is similar to that of the Veneziano model except that in the DAMA model multiple poles appear on daughter levels [3],

$$D(s, t) = \sum_{n=0}^{\infty} D_n(s, t) = \sum_{n=0}^{\infty} g^{n+1} \sum_{l=0}^n \frac{[-s\alpha'_s(s)]^l C_{n-l}(t)}{[n - \alpha_s(s)]^{l+1}}, \quad (9)$$

where $C_n(t)$ is the residue, whose form is fixed by the t -channel Regge trajectory (see [3]). The presence of these multiple poles does not contradict the theoretical postulates. On the other hand, they can be removed without any harm to the dual model by means of the so-called Van der Corput neutralizer [18], corresponding to a slight modification of the integrand in Eq. (7) and resulting in a standard, "Veneziano-like" pole structure [3],

$$D(s, t) = \sum_n g^{n+1} \frac{C_n(t)}{n - \alpha_s(s)}. \quad (10)$$

The pole term in the DAMA model is a generalization of the Breit-Wigner formula, and is equivalent to a sequence of resonances lying on a complex trajectory $\alpha_s(s)$. Such a "Reggeized" Breit-Wigner formula has little practical use in the case of linear trajectories, since it results in an infinite sequence of poles. However, it becomes a powerful tool if complex trajectories are used with a bounded real part and hence a restricted number of resonances. Moreover, it appears that a small number of resonances are sufficient to saturate the direct channel.

In contrast to the Veneziano model, the DAMA amplitude presented in Eq. (7) not only allows the use of nonlinear complex trajectories, but actually requires the presence of such trajectories. More specifically, the asymptotic rise of the trajectories in DAMA is limited by the condition:

$$\left| \frac{\alpha_s(s)}{\sqrt{s} \ln s} \right| \leq \text{const}, \quad s \rightarrow \infty. \quad (11)$$

The condition in Eq. (11) is in accordance with the Froissart bound, an important upper bound on the rate of rise of cross sections at high energy [19]. Actually, the upper bound in Eq. (11) can be lowered up to a logarithm by requiring wide angle scaling behavior for the amplitude.

The boundedness of the real part of the trajectories and, consequently, the termination of resonances lying on it, differs from the general prejudice of their indefinite

rise. The latter is supported by several sources, namely: a) the Veneziano amplitude and the string model; b) the seemingly linear behavior of the spectra of low-lying resonances; c) simplicity. On the other hand, the finite widths of resonances require complex trajectories (as in DAMA). Fits to the masses and decay widths of resonances lying on non-nonlinear trajectories can be found in Ref. [20] (for mesons) and [21] (for baryons). For their physical interpretation in terms of (de)confining quarks in a string model see, e.g., [22, 23] and references therein.

Having introduced DAMA, an important question is to examine the dual properties of the model that incorporates broad resonances, which is a radical modification with respect to the original narrow resonance Veneziano model. This question was studied in detail in Ref. [4], where it was shown that DAMA is dual in the sense that a sum of direct channel resonances reproduces Regge behavior and there is no "double counting" typical of interference models. Moreover, as shown in Ref. [4], a single pole term of DAMA alone may generate Regge behavior.

C. A modified DAMA model

Inclusive e^-p reactions are most efficiently described in terms of the following kinematical variables, the virtuality Q^2 , $Q^2 = -q^2 = -(k - k')^2 \geq 0$ of the exchanged photon and the Bjorken variable $x = Q^2/(2p \cdot q)$ (cf. Fig. 2). Here k and k' are respectively the initial and final lepton momentum and p is the nucleon momentum. The variables x , Q^2 and the Mandelstam variable s (of the γ^*p system) $s = (p + q)^2$, are not independent as they obey the relation:

$$s(x, Q^2) = Q^2(1 - x)/x + m^2, \quad (12)$$

where m is the proton mass. In a recent series of papers

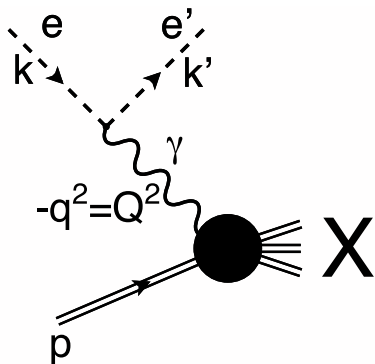


FIG. 2: Kinematics of inclusive lepton-nucleon scattering.

[24–27] attempts were made to build a Q^2 -dependent generalization of the dual amplitude $D(s, t) \rightarrow D(s, t, Q^2)$. This amplitude, a function of three variables, should have correct known limits, *i.e.* it should depend on the on-shell

hadronic scattering amplitude when $Q^2 \sim 0$ on the one hand, and on the nuclear structure function (SF) when $t = 0$, on the other hand. In such a way we hope to complete a unified "two-dimensionally dual" picture of the strong interactions [24–27] as shown schematically in Fig. 1. The first attempts to combine resonance (Regge) behavior with Bjorken scaling were made [28–30] at low energies (large x), with emphasis on the Q^2 -dependence. The amplitudes were chosen such as to match the known behavior of form factors, of vector meson dominance (VMD) with the requirement of Bjorken scaling [46]. In the high-energy (low x) region, such a behavior is supported by HERA data. They are discussed in Sect. III.

While inclusive scattering is determined by the imaginary part in the forward limit, the complete description requires also the knowledge of t dependence. In Ref. [24, 25] the authors attempted to introduce the Q^2 -dependence into the Veneziano amplitude [2] and into the more ambitious Dual Amplitude with Mandelstam Analyticity (DAMA) model [3]. There were some attempts to introduce Q^2 -dependence in the DAMA model either through a Q^2 -dependent Regge trajectory [24], which leads to a problem with its physical interpretation, or through the parameter g [24, 25]. The latter [25] seems more attractive, although it is also restricted by DAMA's intrinsic constraint $g > 1$ [3]. In a series of papers [24–27], the imaginary part of the forward amplitude was related to the total cross section, and to the nucleon SF. In this way, the low- x behavior of the structure function F_2 gives a transcendental equation for $g(Q^2)$ (see [25] for more details). This results in the constraint $g(Q^2 \rightarrow \infty) \rightarrow 0$, inconsistent with the DAMA's constraint $g > 1$. Thus, this procedure at best is valid only for a limited range of Q^2 [25]. An alternative approach is to build a Regge-dual model with Q^2 -dependent form factors [26, 27]. This procedure was inspired by the pole series expansion of DAMA that fits the SF data in the resonance region. The goal was to have a Q^2 -dependent dual amplitude that would lead to such an expansion. The obtained fit was in fairly good agreement with the experimental data, however the attempt to find a general expression for a two-dimensionally dual amplitude $D(s, t, Q^2)$ failed.

Finally, in Ref. [17] a new modified DAMA model with Q^2 -dependence, thereafter referred to as M-DAMA, was proposed. After the DAMA procedure, this constitutes the next step in generalizing the Veneziano model. The M-DAMA model preserves the attractive features of DAMA, such as its pole structure in s and t and Regge asymptotic behavior. An added feature is that its Q^2 -dependent form factors have the correct $Q^2 \rightarrow \infty$ limit when compared with the structure function (at $t = 0$) at large- x . The resulting integral representation for M-DAMA is given by,

$$D(s, t, Q^2) = \int_0^1 dz \left(\frac{z}{g} \right)^{-\alpha_s(s') - \beta(Q^2) - 1}$$

$$\times \left(\frac{1-z}{g} \right)^{-\alpha_t(t'')-\beta(Q'^2)-1}, \quad (13)$$

where $\beta(Q^2)$ is a smooth, dimensionless function of Q^2 which will be further characterized below. The on-mass-shell limit $Q^2 = 0$ leads to a shift of the s - and t -channel trajectories by a constant factor $\beta(0)$. Such a shift can be absorbed into the definition of the Regge trajectories and in this sense the M-DAMA model reduces to DAMA in this limit. In the general case of interaction with a virtual particle with mass M we would have to replace Q^2 by $(Q^2 + M^2)$ in Eq. (13). At this point, all of the machinery developed for DAMA (see for example [3]) can be applied to the above integral. In particular it can be shown [17] that the s -channel pole term in the M-DAMA model has the following expression (*cf.* Eq. (9))

$$D_n(s, t, Q^2) = g^{n+1} \sum_{l=0}^n \frac{[\beta'(0)Q^2 - s\alpha'_s(s)]^l C_{n-l}(t, Q^2)}{[n - \alpha_s(s) - \beta(0)]^{l+1}}. \quad (14)$$

The Q^2 dependence of the coefficients C_l can be directly associated with the form factors and will be further discussed in Sec. IIIB. The presence of the multiple poles, Eq. (14), does not contradict the theoretical postulates. On the other hand, they can be removed without any harm to the dual model by using the Van der Corput neutralizer, as was discussed in Sect. II B. This results in a "Veneziano-like" pole structure:

$$D_n(s, t, Q^2) = g^{n+1} \frac{C_n(t, Q^2)}{n - \alpha_s(s) - \beta(0)}. \quad (15)$$

The asymptotic properties of the M-DAMA model are also similar to those of DAMA. In the Regge limit, ($|s| \rightarrow \infty$, $t, Q^2 = \text{const.}$), the asymptotic behavior of the M-DAMA model is

$$D(s, t, Q^2) \sim s^{\alpha_t(t)+\beta(0)} g^{\beta(Q^2)}, \quad |s| \rightarrow -\infty. \quad (16)$$

Thus, in this limit the M-DAMA model has the same asymptotic behavior as DAMA (except for the shift $\beta(0)$). It is also interesting to consider the regime that does not exist in DAMA, namely the limit $Q^2 \rightarrow \infty$, with constant s, t . If one assumes that $\beta(Q^2) \rightarrow -\infty$ for $Q^2 \rightarrow \infty$, which will be discussed in the following section, then it can be shown that in this limit [17]

$$D(s, t, Q^2) \sim (2g)^{2\beta(Q^2/2)+\alpha_s(s/2)+\alpha_t(t/2)+2}. \quad (17)$$

In the case of deep inelastic scattering (DIS), as we shall see below, if s and t are fixed and $Q^2 \rightarrow \infty$ then $u = -2Q^2 \rightarrow -\infty$, which follows from the kinematic relation $s + t + u = 2m^2 - 2Q^2$. So, we need also to study the amplitude $D(u, t, Q^2)$ in this limit. If $|\alpha(-2Q^2)|$ increases slower than $|\beta(Q^2)|$ or terminates when $Q^2 \rightarrow \infty$, then the previous result (Eq. (17) with s replaced by $u = -2Q^2$) is still valid. We will return to these results in the next section, to check the proposed form of $\beta(Q^2)$.

III. NUCLEON STRUCTURE FUNCTION

Having produced a generalization of the DAMA model to describe virtual photon-nucleon scattering we discuss its implications for the structure functions. Figure 3 shows schematically how inelastic lepton-hadron scattering is related to the forward elastic ($t = 0$) γ^*p amplitude and how the latter can be decomposed into a sum of s -channel resonance exchanges.

A. Scaling behavior

The total cross section for the γ^*p reaction is related to the structure function $F_2(x, Q^2)$ by

$$F_2(x, Q^2) = \frac{Q^2(1-x)}{4\pi\alpha_{em}(1+4m^2x^2/Q^2)} \sigma_t^{\gamma^*p}, \quad (18)$$

where α_{em} is the fine structure constant and we have ignored the longitudinal photon cross section which is a reasonable approximation at high energies. The total cross section can be related to the imaginary part of the scattering amplitude via the optical theorem,

$$\sigma_t^{\gamma^*p}(x, Q^2) = \frac{8\pi}{p\sqrt{s}} \text{Im}A(s, t=0, Q^2), \quad (19)$$

where $p = (s - m^2)\sqrt{1 + 4m^2x^2/Q^2}/2\sqrt{s}(1-x)$. Thus, we have

$$F_2(x, Q^2) = \frac{4Q^2(1-x)^2}{\alpha_{em}(s - m^2)(1 + \frac{4m^2x^2}{Q^2})^{3/2}} \text{Im}A(s, t=0, Q^2). \quad (20)$$

The minimal model for the scattering amplitude with the proper symmetry properties is a sum of s and u -channel amplitudes, [31]

$$A(s, 0, Q^2) = (s - u)(D(s, 0, Q^2) - D(u, 0, Q^2)), \quad (21)$$

which gives the correct signature in the high-energy limit. Note that u is not an independent variable, since $s + u = 2m^2 - 2Q^2$ or $u = -Q^2(1+x)/x + m^2$. As we remarked earlier, we disregard the spin and isospin properties of the amplitude in order to concentrate on the dynamics. In the low- x limit with $t = 0$, and $Q^2 = \text{const}$, $s = Q^2/x \rightarrow \infty$, $u = -s$, we obtain, with the help of Eqs. (16),(21),

$$\text{Im}A(s, 0, Q^2) \rightarrow s^{\alpha_t(0)+\beta(0)+1} g^{\beta(Q^2)}. \quad (22)$$

Our philosophy is then as follows, we specify $\beta(Q^2)$ in this limit and then use the M-DAMA integral representation from Eq. (13) to calculate the dual amplitude, and correspondingly the structure function, in all kinematic domains. We will see that the resulting SF has qualitatively the correct behavior in all regions. Moreover, our

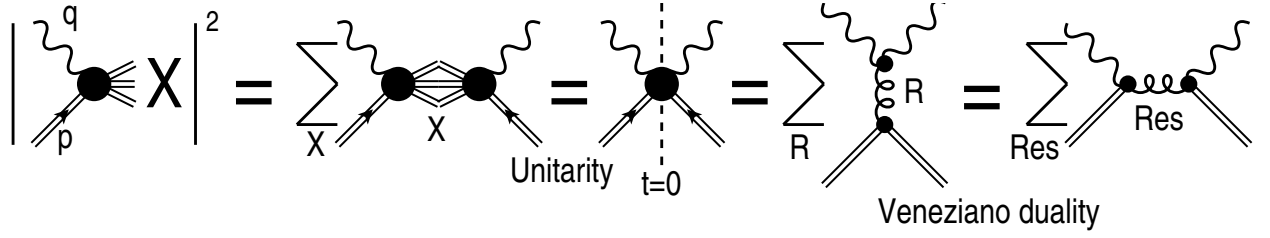


FIG. 3: According to Veneziano, or resonance-Regge, duality a proper sum of either t -channel or s -channel resonance exchanges accounts for the whole amplitude.

choice of $\beta(Q^2)$ will automatically remove the potential possibility of having Q^2 poles [17, 32].

According to the two-component duality [33], both the scattering amplitude A and the structure function F_2 are sums of diffractive and non-diffractive terms. At high energies, both are Regge-behaved. For γ^*p scattering only the positive-signature exchanges are allowed. The dominant contributions come from the Pomeron and the f Reggeon, respectively. The relevant scattering amplitude is therefore as follows:

$$A(s, 0, Q^2) \propto \sum_{k=P,R} \xi_k R_k(Q^2) s^{\alpha_k(0)}, \quad (23)$$

where ξ_k , α_k and R_k are respectively the signature, Regge trajectory and residue. The residue is chosen so as to satisfy approximate Bjorken scaling for the structure function [34, 35]. From Eqs. (20),(23) the SF is given by:

$$F_2(x, Q^2) \propto \sum_{k=P,R} Q^2 R_k(Q^2) s^{\alpha_k(0)-1} \quad (24)$$

where $x = Q^2/s$ in the limit $s \rightarrow \infty$. It is obvious from Eq. (24) that Regge asymptotics and scaling behavior require the residue to fall like $\sim (Q^2)^{-\alpha_k(0)}$. Actually, it could be more involved if we were to account for scaling violation [34–36]. Data show that Pomeron exchange leads to a rising structure function at large s (low x). To provide for this we have two options: either to assume that the Pomeron is supercritical, *i.e.* that $\alpha_P(0) > 1$ or to assume a critical ($\alpha_P(0) = 1$) dipole or higher multipole Pomeron [36–38]. The latter condition leads to logarithmic behavior for the SF [36, 38],

$$F_{2,P}(x, Q^2) \sim Q^2 R_P(Q^2) \ln\left(\frac{s}{m^2}\right). \quad (25)$$

Let us now return to the results of the M-DAMA model. Using Eqs. (20),(22) we obtain,

$$F_2 \sim s^{\alpha_t(0)+\beta(0)} Q^2 g^{\beta(Q^2)}. \quad (26)$$

Choosing $\beta(0) = -1$, we restore the asymptotic condition of Eq. (24), allowing us to use the trajectories in their usual form. It is therefore important to find such a $\beta(Q^2)$ that can provide for Bjorken scaling. A possible

choice for $\beta(Q^2)$ which smoothly connects the large- Q^2 behavior with the low- Q^2 data and has the correct photo-production limit, $\beta(0) = -1$ is given by

$$\begin{aligned} \beta(Q^2) &= \beta(0) - \gamma \ln\left(\frac{Q^2 + Q_0^2}{Q_0^2}\right) \\ &= -1 - \frac{\alpha_t(0)}{\ln g} \ln\left(\frac{Q^2 + Q_0^2}{Q_0^2}\right). \end{aligned} \quad (27)$$

This choice leads to

$$F_2(x, Q^2) \sim x^{1-\alpha(0)} \left(\frac{Q^2}{Q^2 + Q_0^2}\right)^{\alpha_t(0)}, \quad (28)$$

where the slowly varying factor $(Q^2/(Q^2 + Q_0^2))^{\alpha(0)}$ is typical of Bjorken scaling violations (see for example [35]).

Next we consider the large- x limit. As $x \rightarrow 1$ with s fixed, $Q^2 = (s - m^2)/(1 - x) \rightarrow \infty$ and correspondingly $u = -2Q^2 \rightarrow -\infty$. Using Eqs. (17),(20),(21) we find

$$\begin{aligned} F_2(x, Q^2) &\propto (1 - x)^2 Q^4 g^{2\beta(Q^2/2)} \\ &\times \left(g^{\alpha_s(s/2)} - g^{\alpha_u(-Q^2)}\right). \end{aligned} \quad (29)$$

In the limit $Q^2 \rightarrow \infty$, the factors $g^{\alpha_s(s/2)}$ and $g^{\alpha_u(-Q^2)}$ are slowly varying functions of Q^2 under our assumption about $\alpha_u(-Q^2)$. Thus, at large x and large Q^2 the structure function has the power-law behavior

$$F_2(x, Q^2) \propto \left(\frac{2Q_0^2}{Q^2}\right)^{2\gamma \ln 2g} \sim (1 - x)^{2\alpha_t(0) \ln 2g / \ln g}. \quad (30)$$

B. Resonances in the Structure Function

We next consider the resonance region structure function F_2 in the M-DAMA model. The appearance of resonances in the SF at large- x is not surprising by itself, it follows from Eqs. (19) and (20). For simplicity we apply the Van der Corput neutralization procedure that was discussed in Sect. II B. Therefore, the pole terms appear in the form specified in Eq. (15). In the vicinity of the resonance energy $s = s_R$, only the resonance term

$D_R(s, t = 0, Q^2)$ is important in the scattering amplitude and correspondingly in the SF. Using $\beta(Q^2)$ in the form given in Eq. (27), which produces Bjorken scaling at large s , one can obtain the residues at the resonance positions (see Ref. [17] for details):

$$C_1(Q^2) = \left(\frac{gQ_0^2}{Q^2 + Q_0^2} \right)^{\alpha_t(0)} \times \left[\alpha_t(0) + \frac{Q^2}{Q^2 + Q_0^2} \ln g - \frac{\alpha_t(0)}{\ln g} \ln \left(\frac{Q^2 + Q_0^2}{Q_0^2} \right) \right]. \quad (31)$$

The term in front of the bracket provides the typical Q^2 -dependence for the form factor, while the remaining part gives an additional slowly varying dependence on Q^2 . Higher orders of, C_n for subleading resonances, have the same primary Q^2 dependence: the $(Q_0^2/(Q^2 + Q_0^2))^{\alpha(0)}$ form factor. By introducing Q^2 -dependence through the parameter g , we would find, however, a significant difference. As can be seen from Eq. (15), g enters with different powers for different resonances on a single trajectory, with the powers increasing in steps of two. Thus, if $g \sim (Q_0^2/(Q^2 + Q_0^2))^\Delta$, then the form factor for the first ($n = 0$) resonance is proportional to $(Q_0^2/(Q^2 + Q_0^2))^\Delta$, while the second one is proportional to $(Q_0^2/(Q^2 + Q_0^2))^{3\Delta}$, and so on. As discussed in [27] the present accuracy of the data unfortunately does not allow one to discriminate between constant powers of the form factor and increasing ones [39–42].

Let us give a quantitative example of how resonances appear in the structure function. For this simplified discussion we assume that our dual amplitude is a sum of three resonances on some s -channel trajectory, and we also assume a simplified form of the form factor discussed above. From Eq. (15) we find the amplitude proportional to

$$D(s, t = 0, Q^2) \propto \sum_{n=1}^3 g^{n+1} \left(\frac{gQ_0^2}{Q^2 + Q_0^2} \right)^{\alpha_t(0)} \frac{1}{n - \alpha_s(s) + 1}. \quad (32)$$

For the Regge trajectory, for simplicity we assume a linear form with a square root branch point at the pion-nucleon threshold, $s_0 = (m_\pi + m)^2$,

$$\alpha_s(s) = \alpha_s(0) + \alpha'_s(0)s + \gamma(\sqrt{s_0} - \sqrt{s_0 - s}), \quad (33)$$

with typical values $\alpha_s(0) = 0.1$, $\alpha'_s(0) = 1 \text{ GeV}^{-2}$ and $\gamma = 0.1 \text{ GeV}^{-1}$ for the intercept and slope of the trajectory and width of the resonances, respectively [47].

Other parameters of the model are: $\alpha_t(0) = 0.5$, $Q_0^2 = 1 \text{ GeV}^2$, and $g = 1.5$. This choice of the scale parameter g gives three resonance peaks of similar height in $\text{Im } D(s, t = 0, Q^2)$ as shown in Fig. 4. We show how the positions of the resonances move as a function of Q^2 and x in Fig. 5 and 6. In Fig. 5 we plot the imaginary part of the dual amplitude $D(s, t = 0, Q^2)$ as a function of either

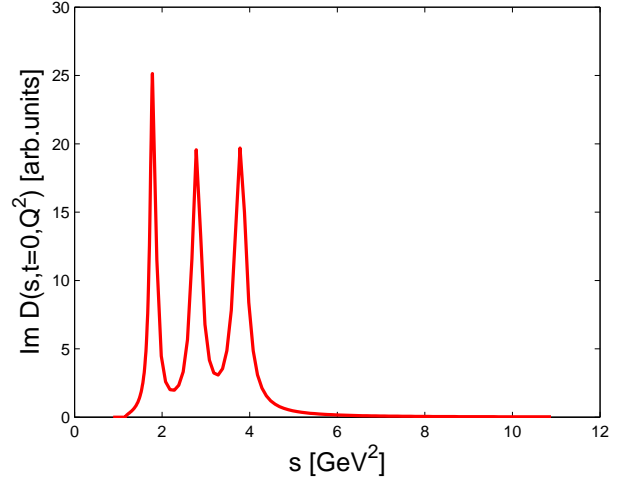


FIG. 4: (color online) Imaginary part of the dual amplitude (in arbitrary units) of Eq. (32) as a function of the Mandelstam variable s .

x or Q^2 . The 2-dimensional plot in Fig. 6 clearly shows how the positions of the resonances move according to (cf. Eq. (12)),

$$s_{R,n} = Q^2(1 - x)/x + m^2 = \text{const.} \quad (34)$$

Similar plots for the structure function $F_2(x, Q^2)$ can be produced. We use Eqs. (20) and (21), and we neglect the u, t terms in Eq. (21), which is justified in the domain where the s -channel resonances dominate. In Fig. 7 we plot the structure function $F_2(x, Q^2)$, as a function of x and Q^2 and in Fig. 8 we plot various cuts on the SF $F_2(x, Q^2)$. We see that, with increasing Q^2 , the resonances produce a power-law behavior at large x .

Some of the qualitative features of the structure function $F_2(x, Q^2)$ are analogous to those for the dual amplitude shown in Figs. 6 and 4. With increasing Q^2 , the resonance peaks move to higher values of x and their spacing in x also increases. Also, with increasing x the resonances move to higher Q^2 and their spacing in Q^2 increases, as is shown in Fig. 8. However, while the three resonance peaks in the dual amplitude have nearly equal heights, the resonance peaks in the SF F_2 have rather different heights, which is evident by comparing Figs. 4 and 8.

IV. SUMMARY

Fig. 1 schematically shows the "correspondence principle", partly realized in our model. The dual amplitude should reduce to a sum of resonance poles or smooth functions scaling in x in the relevant kinematical regions, and it should smoothly interpolate between these two regions

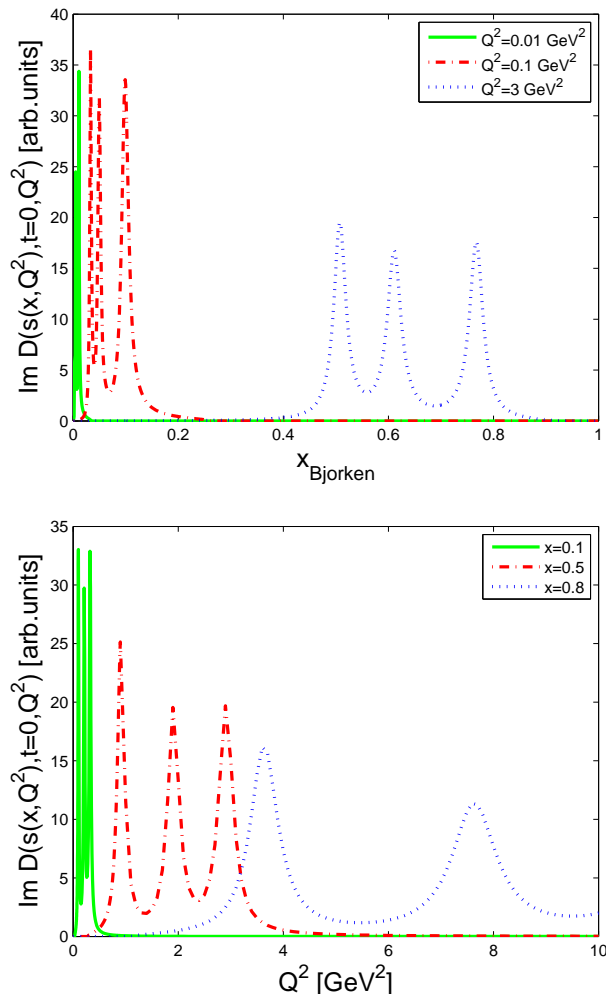


FIG. 5: (color online) Properties of the dual amplitude, given by Eq. (32) as a function of Bjorken x (top panel) for various values of Q^2 and as a function of the virtuality Q^2 for various values of x (lower panel).

along the vertical line of the lower icon of Fig. 1. The upper icon of the same figure shows this effect on the large- x behavior of the structure function.

In this paper we have presented an explicit model that realizes parton-hadron duality. In the present model, the same amplitude interpolates between low- and high x , and thus realizes “Veneziano duality”, or resonance-Regge duality at low Q^2 . The M-DAMA model exhibits “Bloom-Gilman” or parton-hadron duality, interpolating between resonant behavior at low Q^2 and power-law behavior at high Q^2 in the large- x region. In this way, the M-DAMA model realizes “two dimensional duality.”

The M-DAMA model, with three resonances approximating the entire resonance regime is a simple pedagogical example. Nevertheless, the model successfully incorporates two types of duality (two-dimensional duality).

Can the path shown schematically in Fig. 1 be closed by relating small- and large- x with some (any?) values of Q^2 ? In other words, given the off-mass-shell dual amplitude in the M-DAMA model that interpolates between the above regions, one can ask if the large- x structure function “knows about” its small- x behavior and vice versa? This would require very strong correlations between the structure function in different regions. We do not have any convincing answer to this question but it is an interesting conjecture given the success of the M-DAMA model that we present in this paper.

The notions of duality can be formulated and applied in a rigorous fashion. Note that one consequence of duality means that a scattering amplitude cannot be decomposed into a sum of resonances plus an independent smooth asymptotic term, as is the case in interference models. This criterion was studied in detail in Ref. [4], where the DAMA model was shown to have genuine dual properties. It would be interesting to see if a similar criterion and proof for parton-hadron duality could be obtained for the M-DAMA model.

One of the main virtues of the M-DAMA model is its applicability to physical processes over a wide kinematic region and connections imposed by the duality conditions. Detailed quantitative applications of this model will require the imposition of various constraints on the amplitudes. Our model utilizes two-component duality, in which the scattering amplitude is a sum of a diffractive and non-diffractive term. In the first case, the high-energy behavior is determined by the exchange of a vacuum (Pomeron) trajectory that is dual to the direct-channel exotic trajectory. The ordinary s and t channel trajectories are connected with resonance spectra. The trajectories, both leading and non-leading, are strongly bounded non-linear complex functions. Examples of practical applications can be found in Refs.[16, 25–27].

The model is not unique, although the form of the dual amplitude is quite constrained as been discussed above, see also [3, 4, 17]. The remaining freedom can be further narrowed by fitting the model to experimental data. Furthermore, one can compare the model to other approaches to high-energy reactions, such as the quark counting rules (in the hard region [27]), string models e.g. in the large- x region [43], or QCD. In this way one can hope to reduce some of the freedom in our dual model and to constrain its free parameters.

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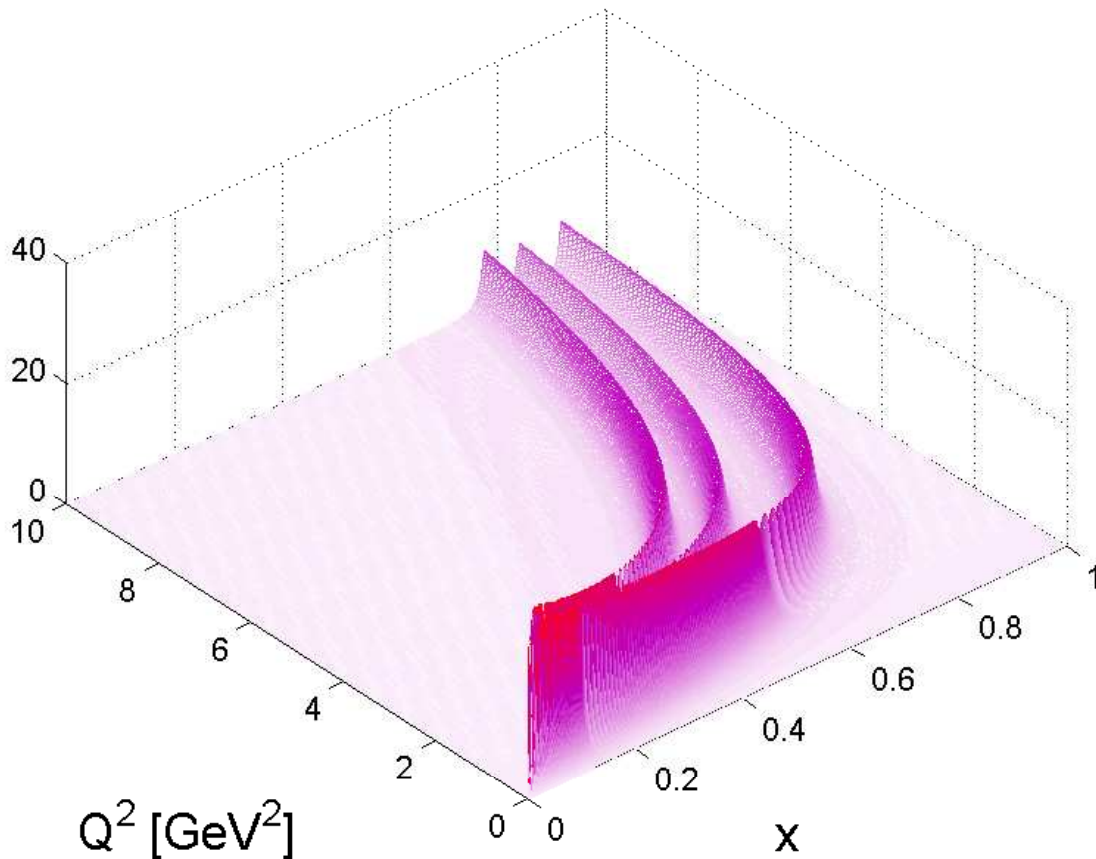


FIG. 6: (color online) Position of the three resonances in the imaginary part of the dual amplitude on the $x - Q^2$ plot.

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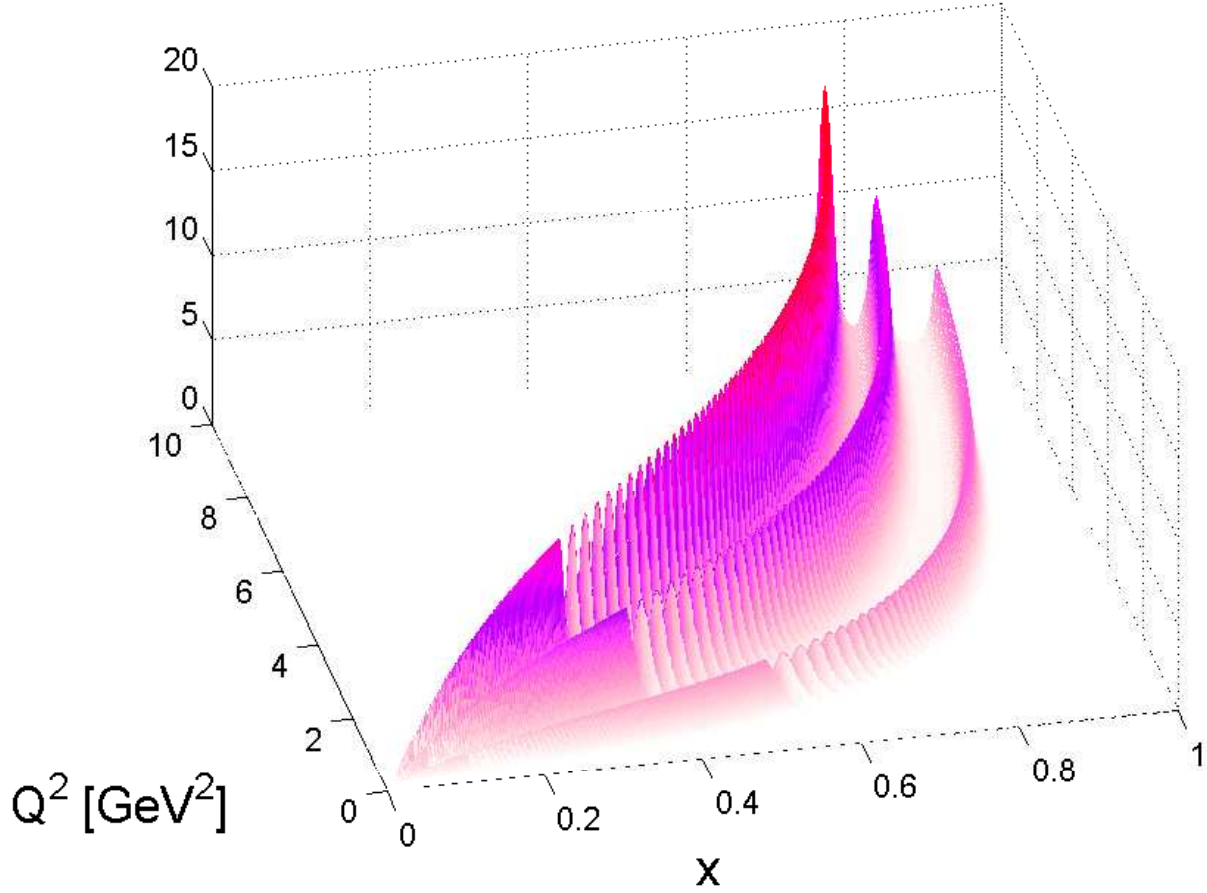


FIG. 7: (color online) The structure function F_2 (in arbitrary units) as a function of Q^2 and x .

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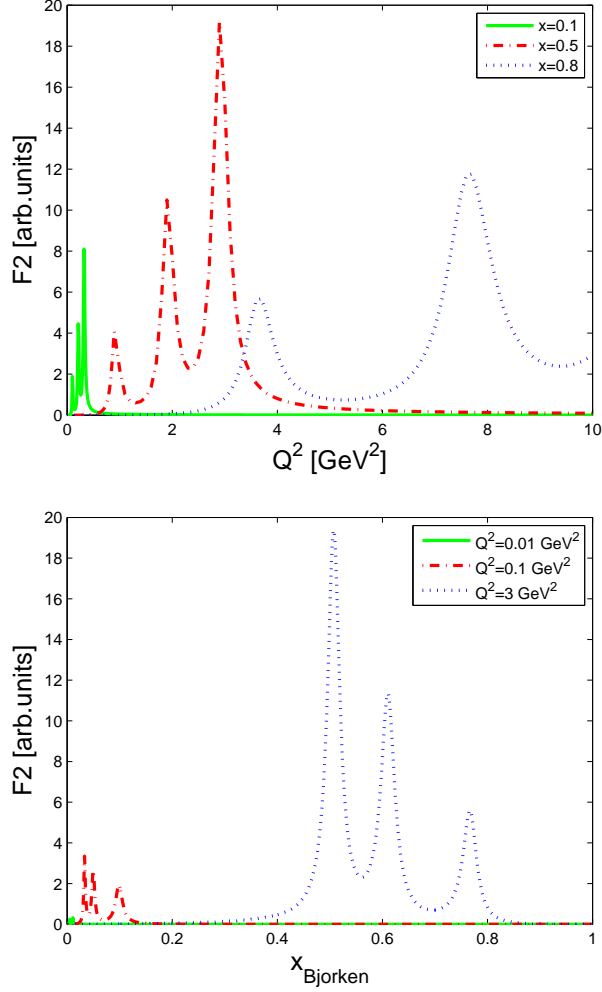


FIG. 8: (color online) Properties of the structure function (SF) $F_2(x, Q^2)$ of Fig. 6 for various kinematics, as a function of Q^2 (top panel) and as a function of Bjorken x (lower panel).

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